$$\int k \cdot f(x)dx = k \cdot \int f(x)dx$$

$$\int [f(x) + g(x) + p(x)]dx = \int f(x)dx + \int g(x)dx + \int p(x)dx$$

Integrali indefiniti immediati

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad n \neq -1 \qquad \int [f(x)]^n \cdot f'(x)dx = \int [f(x)]^n \cdot df(x) = \frac{[f(x)]^{n+1}}{n+1} + C$$

$$\int \frac{1}{x}dx = \ln|x| + C \qquad \int \frac{f'(x)}{f(x)} \cdot dx = \int \frac{1}{f(x)} \cdot d\,f(x) = \ln|f(x)| + C$$

$$\int \cos x \cdot dx = \sin x + C \qquad \int f'(x)\cos f(x) \cdot dx = \int \cos f(x) \cdot df(x) = \sin f(x) + C$$

$$\int \sin x \cdot dx = -\cos x + C \qquad \int f'(x)\sin f(x) \cdot dx = \int \sin f(x) \cdot df(x) = -\cos f(x) + C$$

$$\int \frac{1}{\cos^2 x}dx = tg\,x + C \qquad \int \frac{f'(x)}{\cos^2 f(x)}dx = \int \frac{d\,f(x)}{\cos^2 f(x)} = tg\,f(x) + C$$

$$\int \frac{1}{\sin^2 x}dx = -\cot g\,x + C \qquad \int \frac{f'(x)}{\sin^2 f(x)}dx = \int \frac{d\,f(x)}{\sin^2 f(x)} = -\cot g\,f(x) + C$$

$$\int e^x dx = e^x + C \qquad \int f'(x)e^{f(x)} dx = \int e^{f(x)} d\,f(x) = e^{f(x)} + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \qquad \int f'(x)a^{f(x)} dx = \int a^{f(x)} d\,f(x) = \frac{a^{f(x)}}{\ln a} + C$$

$$\int \frac{1}{1+x^2}dx = \text{arctg}\,x + C \qquad \int \frac{f'(x)}{1+[f(x)]^2}dx = \int \frac{d\,f(x)}{1+[f(x)]^2} = \text{arctg}\,f(x) + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C \qquad \int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}dx = \int \frac{d\,f(x)}{\sqrt{1-[f(x)]^2}} = \arcsin f(x) + C$$

$$\int \frac{1}{\sin x}d\,x = \ln\left|tg\frac{x}{2}\right| + C \qquad \int \frac{1}{\sin f(x)}d\,f(x) = \ln\left|tg\frac{f(x)}{2}\right| + C$$

$$\int \frac{1}{\cos x}d\,x = \ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C \qquad \int \frac{1}{\cos f(x)}d\,f(x) = \ln\left|tg\left[\frac{f(x)}{2}+\frac{\pi}{4}\right]\right| + C$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}d\,x = \ln\left|x+\sqrt{x^2 \pm a^2}\right| + C \qquad \int \frac{1}{\sqrt{[f(x)]^2 \pm a^2}}d\,f(x) = \ln\left|x+\sqrt{[f(x)]^2 \pm a^2}\right| + C$$

$$\int \frac{1}{a^2+x^2}d\,x = \frac{1}{a}\text{arctg}\frac{x}{a} + C \qquad \int \frac{1}{a^2+[f(x)]^2}d\,f(x) = \frac{1}{a}\text{arctg}\frac{f(x)}{a} + C$$

$\int \sinh x \cdot dx = \cosh x + C$ $\int \sinh f(x) \cdot d\, f(x) = \cosh f(x) + C$

$\int \cosh x \cdot dx = \sinh x + C$ $\int \cosh f(x) \cdot d\, f(x) = \sinh f(x) + C$

$\int \frac{1}{\sqrt{1+x^2}} dx = \text{settsinh}\, x + C = \ln\left|x + \sqrt{1+x^2}\right|$ $\int \frac{1}{\sqrt{1+[f(x)]^2}} d\, f(x) = \text{settsinh}\, f(x) = \ln\left|x + \sqrt{1+[f(x)]^2}\right|$

Integrazione per parti

$$\int \varphi(x)dx = \int f(x) \cdot \underline{g'(x)dx} = \int f(x)\underline{d\, g(x)} = f(x)g(x) - \int g(x)df(x) =$$

$$= f(x)g(x) - \int g(x) f'(x)dx \qquad \textbf{[7]}$$

Integrazione per sostituzione

$$F(x) + C = \int f(x)dx = \int f[g(t)] \cdot g'(t) \cdot dt = F[g(t)] + C \quad x = g(t) \quad dx = g'(t)dt$$

Integrazione per sostituzione: metodi generali

$\int R\left(x, \sqrt{\frac{x}{a-x}}\right) dx$ $x = a \cdot \sin^2 t$ $dx = 2a \cdot \sin t \cdot \cos t \cdot dt$ $\sqrt{\frac{x}{a-x}} = tg\, t$

$\int R\left(x, \sqrt{a^2 - x^2}\right) dx$ $x = a \cdot \sin t$ $dx = a \cdot \cos t \cdot dt$ $a^2 - x^2 = a^2 \cdot \cos^2 t$

$\int \mathbf{R}\left(\mathbf{x}, \sqrt{\mathbf{a}^2 + \mathbf{x}^2}\right) \mathbf{dx}$ $x = a \cdot \text{tg}\, t$ $x = a \cdot \text{settsh}\, t$

$\int \mathbf{R}\left(\mathbf{x}, \sqrt{\mathbf{x}^2 - \mathbf{a}^2}\right) \mathbf{dx}$ $x = \frac{a}{\cos t}$ $x = a \cdot \text{settch}\, t$

$\int \mathbf{R}\left(\mathbf{x}, \sqrt{\mathbf{x}}, \sqrt{\mathbf{1-x}}\right) \mathbf{dx}$ $x = \sin^2 t$ $dx = 2 \sin t \cdot \cos t \cdot dt$

$\int \sqrt{\mathbf{a}^2 - \mathbf{x}^2}\, \mathbf{dx}$ $x = a \cdot \sin t$ $dx = a \cdot \cos t \cdot dt$ $a^2 - x^2 = a^2 \cdot \cos^2 t$ oppure $x = \text{tgh}\, x$

$\int \frac{\mathbf{1}}{\sqrt{\mathbf{a}^2 - \mathbf{x}^2}} \mathbf{dx}$ $x = a \cdot \sin t$ $dx = a \cdot \cos t \cdot dt$ $a^2 - x^2 = a^2 \cdot \cos^2 t$

$\int \sqrt{\mathbf{x}^2 - \mathbf{a}^2}\, \mathbf{dx}$ $x = \frac{a}{\cos t}$ $x = a \cdot \cosh t$

$\int \frac{\mathbf{1}}{\sqrt{\mathbf{x}^2 - \mathbf{a}^2}} \mathbf{dx}$ $x = \frac{a}{\cos t}$ $x = a \cdot \cosh t$

$\int \sqrt{a^2 + x^2}\,dx \quad x = a \cdot \operatorname{tg} t \quad x = a \cdot \sinh t$

$\int \frac{1}{\sqrt{a^2 + x^2}}\,dx \quad x = a \cdot \operatorname{tg} t \quad x = a \cdot \sinh t$

$\int \sqrt{ax^2 + bx + c}\,dx$ $\int R\left(x, \sqrt{ax^2 + bx + c}\right)dx$ possiamo utilizzare una delle tre seguenti **sostituzioni di Eulero**.

Le tre sostituzioni di Eulero in sintesi

$a > 0$	$\sqrt{ax^2 + bx + c} = t \pm \sqrt{a} \cdot x$ oppure $\sqrt{ax^2 + bx + c} = \sqrt{a} \cdot (t \pm x)$
$c > 0$	$\sqrt{ax^2 + bx + c} = xt \pm \sqrt{c}$
$\Delta = b^2 - 4ac > 0$	$\sqrt{ax^2 + bx + c} = \sqrt{a(x - \alpha)(x - \beta)} = (x - \alpha)t$

Prima sostituzione di Eulero

1° caso **$a > 0$** c **qualsiasi** **$\Delta = b^2 - 4ac > 0$** $\sqrt{ax^2 + bx + c} = \sqrt{a(x - \alpha)(x - \beta)}$

L'integrale si risolve utilizzando una delle seguenti sostituzioni:

$$\sqrt{ax^2 + bx + c} = \begin{cases} t \pm \sqrt{a} \cdot x & \text{oppure} \\ \sqrt{a} \cdot (t \pm x) & \text{oppure} \\ (x - \alpha)t \end{cases}$$ **sostituzione algebrica**

$$D\left(ax^2 + bx + c\right) = 2ax + b = \begin{cases} \sqrt{\Delta} \cdot \sec t = \frac{\sqrt{\Delta}}{\cos t} & \text{oppure} \\ \sqrt{\Delta} \times \operatorname{cosec} t = \frac{\sqrt{\Delta}}{\sin t} \end{cases}$$ **sostituzione goniometrica**

Seconda sostituzione di Eulero

2° caso **$a > 0$** **$\Delta = b^2 - 4ac < 0$**

$$\sqrt{ax^2 + bx + c} = \begin{cases} t \pm \sqrt{a} \cdot x & \text{oppure} \\ \sqrt{a} \cdot (t \pm x) & \text{oppure} \\ tx \pm \sqrt{c} & \text{se } c > 0 \end{cases}$$ **sostituzione algebrica**

$$D\left(ax^2 + bx + c\right) = 2ax + b = \begin{cases} \sqrt{-\Delta} \cdot \operatorname{tg} t & \text{oppure} \\ \sqrt{-\Delta} \cdot \operatorname{cotg} t \end{cases}$$ **sostituzione goniometrica**

Terza sostituzione di Eulero

3° caso $\quad a < 0 \quad \Delta = b^2 - 4ac > 0 \quad \sqrt{ax^2+bx+c} = \sqrt{a(x-\alpha)(x-\beta)}$

$$\sqrt{ax^2 + bx + c} = \begin{cases} (x-\alpha)t & \text{oppure} \\ (\beta - x)t \end{cases}$$ sostituzione algebrica

$$D(ax^2+bx+c) = 2ax+b = \begin{cases} \sqrt{\Delta} \times \sin t & \text{oppure} \\ \sqrt{\Delta} \times \cos t \end{cases}$$ sostituzione goniometrica

$$\int R(\sin^2 x, \cos^2 x, tg\, x) \cdot dx \quad tg\, x = t \quad x = \operatorname{arctg} t \quad dx = \frac{1}{1+t^2} dt \quad \sin^2 x = \frac{t^2}{1+t^2} \quad \cos^2 x = \frac{1}{1+t^2}$$

$$\int R(\sin x, \cos x, tg\, x) dx \quad tg \frac{x}{2} = t \quad \frac{x}{2} = \operatorname{arctg} t \quad x = 2 \operatorname{arctg} t \quad dx = \frac{2}{1+t^2} dt$$

$$\sin x = \frac{2 tg \frac{x}{2}}{1 + tg^2 \frac{x}{2}} = \frac{2t}{1+t^2} \qquad \cos x = \frac{1 - tg^2 \frac{x}{2}}{1 + tg^2 \frac{x}{2}} = \frac{1-t^2}{1+t^2} \qquad tg\, x = \frac{2 \cdot tg \frac{x}{2}}{1 - tg^2 \frac{x}{2}} = \frac{2t}{1-t^2}$$

- $\int \sin^{2n} x \cdot dx = \int (\sin^2 x)^n \cdot dx = \int \left(\frac{1 - \cos 2x}{2} \right)^n \cdot dx$
- $\int \sin^{2n+1} x \cdot dx = \int (\sin^2 x)^n \cdot \sin x \cdot dx = -\int (1 - \cos^2 x)^n \cdot d \cos x$
- $\int \cos^{2n} x \cdot dx = \int (\cos^2 x)^n \cdot dx = \int \left(\frac{1 + \cos 2x}{2} \right)^n \cdot dx$
- $\int \cos^{2n+1} x \cdot dx = \int (\cos^2 x)^n \cdot \cos x \cdot dx = \int (1 - \sin^2 x)^n \cdot d \sin x$
- $\int R(e^{\alpha x}) dx \qquad \int R(e^{\alpha x}) dx = \frac{1}{\alpha} \int \frac{R(t)}{t} dt \qquad e^{\alpha x} = t, \quad x = \frac{\ln t}{\alpha}, \quad dx = \frac{dt}{\alpha t}$
- $\int P(x) \cdot \underline{e^{\alpha x} dx} = \frac{1}{\alpha} P(x) e^{\alpha x} - \frac{1}{\alpha} \int P'(x) \underline{e^{\alpha x} dx}$ $\quad P(x)$ polinomio razionale intero di grado n
- $\int R(\ln x) dx \qquad \ln x = t \qquad x = e^t \qquad dx = e^t dx$